

## The Matrix: Applying Matrices to Explore Plant Life Histories

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1. Building the matrix model. Suppose that a shrub recently colonized a new habitat in the desert with a life history that evolved in a different environment. We shall assume that seeds and adults are key stages in the life of this shrub. The life history traits of this shrub species are:

Seeds produced per adult plant per year (seed production) = 80

Proportion of seeds germinating per year (shrub recruitment) = 0.02 or 2%

Proportion of seeds surviving without germinating per year (seed survival) = 0.05 or 5%

Proportion of adult plants surviving each year (shrub survival) = 0.9 or 90%

Suppose that the population starts with  $x_0$  seeds and  $y_0$  shrubs (year  $t = 0$ ), write a linear equation that gives the number of seeds for the next year  $x_1$  (year  $t = 1$ ) as a function of  $x_0$  and  $y_0$ . Also write the linear equation for  $y_1$  as a function of  $x_0$  and  $y_0$

$$x_1 = \underline{\hspace{10em}}$$

$$y_1 = \underline{\hspace{10em}}$$

Express the equations above as a matrix equation.

$$\begin{bmatrix} \underline{\hspace{1em}} \\ \underline{\hspace{1em}} \end{bmatrix} = \begin{bmatrix} \underline{\hspace{1em}} & \underline{\hspace{1em}} \\ \underline{\hspace{1em}} & \underline{\hspace{1em}} \end{bmatrix} \begin{bmatrix} \underline{\hspace{1em}} \\ \underline{\hspace{1em}} \end{bmatrix}$$

where  $x_t$  is the number of seeds and  $y_t$  is the number of shrubs at year  $t$ .

2. Suppose that we start a new population of this species with only one shrub. Compute the number of seeds and the number of adult shrubs in the population for 6 generations. Use a graphing calculator in your computations.

Year 1

$$\begin{bmatrix} \_ \\ \_ \end{bmatrix} = \begin{bmatrix} 0.05 & 80 \\ 0.02 & 0.9 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Total  $\_$

Year 2

$$\begin{bmatrix} \_ \\ \_ \end{bmatrix} = \begin{bmatrix} 0.05 & 80 \\ 0.02 & 0.9 \end{bmatrix} \begin{bmatrix} \_ \\ \_ \end{bmatrix}$$

Total  $\_$

Year 3

$$\begin{bmatrix} \_ \\ \_ \end{bmatrix} = \begin{bmatrix} 0.05 & 80 \\ 0.02 & 0.9 \end{bmatrix} \begin{bmatrix} \_ \\ \_ \end{bmatrix}$$

Total  $\_$

Year 4

$$\begin{bmatrix} \_ \\ \_ \end{bmatrix} = \begin{bmatrix} 0.05 & 80 \\ 0.02 & 0.9 \end{bmatrix} \begin{bmatrix} \_ \\ \_ \end{bmatrix}$$

Total  $\_$

Year 5

$$\begin{bmatrix} \_ \\ \_ \end{bmatrix} = \begin{bmatrix} 0.05 & 80 \\ 0.02 & 0.9 \end{bmatrix} \begin{bmatrix} \_ \\ \_ \end{bmatrix}$$

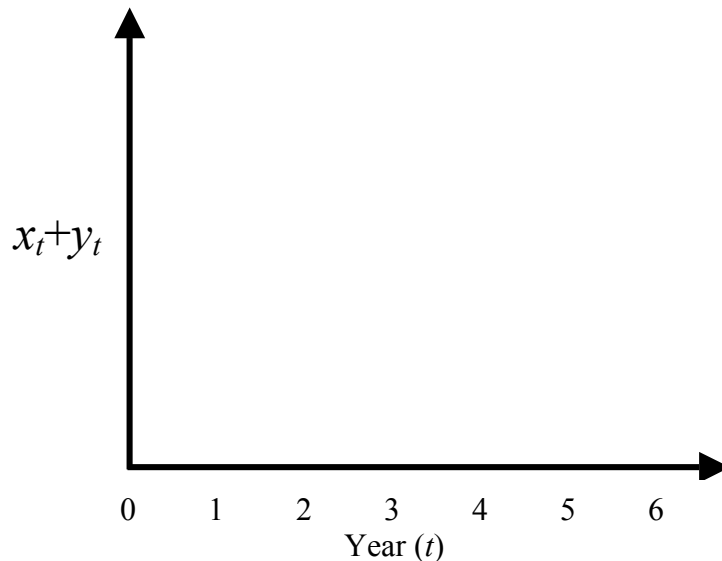
Total  $\_$

Year 6

$$\begin{bmatrix} \_ \\ \_ \end{bmatrix} = \begin{bmatrix} 0.05 & 80 \\ 0.02 & 0.9 \end{bmatrix} \begin{bmatrix} \_ \\ \_ \end{bmatrix}$$

Total  $\_$

Sketch a plot of the total population size (number of seeds plus number of shrubs) in the vertical axis and year (0, 1, ..., 6) in the horizontal axis.



### 3. Application of matrices to the study of the evolution of life history traits.

Natural populations cannot grow forever as predicted by our matrix model because of a limited resource supply in nature and many other factors. From the model matrix above, you can try a different set of values for seed and adult survival and seed production using programming functions using graphing calculators or open source software such as

*R* (<http://www.r-project.org/>) or *Populus* (available at <http://www.cbs.umn.edu/populus/>).

Change the value of one parameter at a time. Examples using the TI-84 graphing calculator and Populus 5.4 are provided below.

#### TI-84 Plus Graphing Calculator

1. Create a  $2 \times 2$  matrix A in the matrix editor. Matrix A will be the population matrix containing seed survival and production, and shrub survival and recruitment.
2. Create a  $2 \times 1$  matrix B in the matrix editor. Matrix B will contain the number of seeds (first entry) and the number of shrubs (second entry).
3. Download or write the following program

Command	Command function
PROGRAM:MATMOD1	Program name
:DelVar L <sub>1</sub>	Deletes list L <sub>1</sub> from previous use
:DelVar L <sub>2</sub>	Deletes list L <sub>2</sub> from previous use
:Disp "SEED SURVIVAL"	Screen display
:Prompt S	Inputs seed survival value
:Disp "SEED PRODUCTION"	Screen display
:Prompt P	Inputs seed production value
:Disp "SHRUB RECRUITMENT"	Screen display
:Prompt R	Inputs shrub recruitment value
:Disp "SHRUB SURVIVAL"	Screen display
:Prompt Q	Inputs shrub survival value
:Disp "GENERATIONS"	Screen display
:Prompt T	Inputs number of generations

:S→[A] (1, 1)	Stores seed survival in $a_{11}$
:P→[A] (1, 2)	Stores seed production in $a_{12}$
:R→[A] (2, 1)	Stores shrub recruitment in $a_{21}$
:Q→[A] (2, 2)	Stores shrub survival in $a_{22}$
:Disp "INIT. POP. SIZE"	Screen display
:Prompt X, Y	Inputs initial number of shrubs and seeds
:X→[B] (1, 1)	Stores initial number of seeds in $b_{11}$
:Y→[B] (2, 1)	Stores initial number of shrubs in $b_{21}$
:0→L <sub>1</sub> (1)	Stores initial generation as "0" in list L <sub>1</sub>
:X+Y→L <sub>2</sub> (1)	Stores initial population size in list L <sub>2</sub>
:For (K, 1, T+1, 1)	Sets a repetitive computations to calculate
: [A] * [B] → [B]	Computes the
: [B] (1, 1) + [B] (2, 1) →N	Computes the sum of shrubs and seeds ( $N$ )
:K→L <sub>1</sub> (K+1)	Stores the generation in list L <sub>1</sub>
:N→L <sub>2</sub> (K+1)	Stores $N$ in list L <sub>2</sub>
:End	Ends computation over $T$ generations
:Plot1 (Scatter, L <sub>1</sub> , L <sub>2</sub> , °)	Plots population size changes over time
:L <sub>1</sub> (1) →ZXmin	Sets zooming lower limit for $T=0$
:max (L <sub>1</sub> ) →ZXmax	Sets zooming upper limit for $T=K$
:0→ZYmin	Sets zooming lower limit for $N=0$
:max (L <sub>2</sub> ) →ZYmax	Sets zooming upper level for the maximum $N$
:ZoomStat	Zooms plot to the specified limits
:DispGraph	Displays $N$ vs. $T$ plot