Computational Thinking and Ecosystem Modeling in High School Biology Classes

A Lesson in Plant Competition
Lesson objectives

• Students will be able to identify mechanisms through which two plant species coexist.
• Students will use mathematical models to address key ecological concepts in plant ecology.
• Students will learn to apply a mathematical models to ecological examples.
National Science Standards: 
*The Interdependence of Organisms*

Organisms both cooperate and compete in ecosystems. The interrelationships and interdependencies of these organisms may generate ecosystems that are stable for hundreds or thousands of years.

Living organisms have the capacity to produce populations of infinite size, but environments and resources are finite. This fundamental tension has profound effects on the interactions between organisms.
Ecology

- Ecology is the study of the factors that determine the abundance and distribution of the organisms
  - Why a plant is common in a certain location and absent from another?
Concepts

- Community
  - Assemblage of populations of living organisms in a prescribed area.

http://en.wikipedia.org
Community Ecology

... studies the mechanisms by which organisms from different species form assemblages or associations.
How do species interact in a community?

Predation
Parasitism
Mutualism
Competition
How do species interact in a community?

Competition

Two organisms use the same limited resource, or seek that resource, to the detriment of both.
Principle of competitive exclusion  
(or Gause’s hypothesis)

Two species competing for the same resources cannot stably coexist 

or 

Two species with similar ecology cannot live together in the same place
Principle of competitive exclusion
Principle of competitive exclusion
Biological invasions: Buffelgrass

http://www.buffelgrass.org/

http://www.youtube.com/watch?v=nQtlVzSrqZY
Therefore, how can species coexist?
Computational Thinking

We will devise a mechanism by which competition allows coexistence of two plant species in a community using computational thinking.
Computational Thinking

- Computational thinking (CT) involves solving problems, and understanding system’s behavior, by drawing on the concepts fundamental to computer science. (Wing 2006)
Modeling competition
Modeling competition

- Suppose that we have two plant species that compete for water, nutrients, space, etc.
Describing competition in mathematical terms

• Step 1
  Find an equation for the population growth of a plant species alone that considers:
  ✓ Reproduction
  ✓ How competition reduces reproduction
Describing competition in mathematical terms

• Step 2
Find an equation for the growth of a plant population in the presence of another species that considers:
✓ How the presence of the other species reduces its reproduction
Function

- Function
  - A rule of correspondence between two sets of numbers
  - For example,
    \[ s(t) = \frac{1}{t} \]
    for all \( t = 1, 2, \ldots \)
    is a rule of correspondence between the set of rational numbers and the set of positive integers.
Graphic representation

\[ s(t) = \frac{1}{t} \]
Modeling competition

• Notation
  \( x(t) \) : number of plants for species \( x \) at year \( t \)
  \( y(t) \) : number of plants for species \( y \) at year \( t \)

• Short notation
  \( x(t) = x_t \)
  \( y(t) = y_t \)
Modeling plant competition

- $x_0$, $x_1$, $x_2$, $x_3$, $x_4$
- Number of plants
- Years
- $n$

Graph showing the number of plants over years.
Change in population size after year $t$ ($\Delta x_t$)
Increment change in population size after year $n$ ($\Delta x_n$)

$\Delta x_3 = x_4 - x_3$
Population growth ($\Delta x_n$)
Population growth ($\Delta x_n$)

- Population growth ($\Delta x_n$)
- $x_n$
- $t$
- Number of plants
- $\Delta x_0 = 0$
- Years
- 0 1 2 3 4
- $x_0$ $x_1$ $x_2$ $x_3$ $x_4$
Population growth ($\Delta x_n$)
Population growth ($\Delta x_n$)
Population growth ($\Delta x_n$)

Years

Number of plants

$\Delta x_3 > 0$
How does a plant population grow in the absence of another species?

Population growth only depends on population size:

- **Each plant produces “r” new plants each year**
- **More plants mean less resources**
Symbols

\( r_x \)  Number of new plants produced by plant of species \( x \) per year in the absence of any other plant

\( \Delta x_t \)  Number of plants produced by all plants of species \( x \) at year \( t \)
How does a plant population grow in the absence of another species?

Each plant produces $r_x$ plants per year

$$\Delta X_t =$$
How does a plant population grow in the absence of another species?

Each plant produces $r_x$ plants per year

$$\Delta x_t = r_x x_t$$

and more plants means less resources

$$\Delta x_t = r_x x_t(\ )$$
How does a plant population grow in the absence of another species?

Each plant produces $r_x$ plants per year

$$\Delta x_t = r_x x_t$$

and more plants mean less resources

$$\Delta x_t = r_x x_t \left(1 - c_{xx} x_t\right)$$
How does a plant population grow in the absence of another species?

\[ \Delta x_t = r_x x_t (1 - c_{xx} x_t) \]
How does a plant population grow in the absence of another species?

\[ \Delta x_t = r_x x_t (1 - c_{xx} x_t) \]

\[ x_{t+1} - x_t = r_x x_t (1 - c_{xx} x_t) \]

\[ x_{t+1} = r_x x_t (1 - c_{xx} x_t) + x_t \]

Recursion
Concept

• Recursion
  – “Computation of a succession of numbers by operation on one or more preceding elements according to a formula” (Merriam-Webster Dictionary)
  – For example,

\[ x_1 = r_x x_0 \left( 1 - c_{xx} x_0 \right) + x_0 \]

The formula implies that to calculate \( x \) at year 1 we need to be given an initial population size \( x_0 \).
Recursion

\[ x_{t+1} = r_x x_t (1 - c_{xx} x_t) + x_t \]

Let’s see what kind of growth this plant population has in Excel.
Modeling in MS Excel

Set up a spreadsheet for 30 years with 10 plants as the initial population:

\[ x_0 = 10 \]
\[ r_x = 0.5 \]
\[ c_{xx} = 0.01 \]
Modeling in MS Excel

Let’s type the equation

\[ x_{t+1} = 0.5x_t(1 - 0.01x_t) + x_t \]

This equation is typed in cell B3 as

=0.5*B2*(1-0.01*B2)+B2
Modeling in MS Excel

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>14.5</td>
</tr>
</tbody>
</table>

To calculate the sequence of values $x_2, x_3, x_4, \ldots, x_{30}$, we just drag the formula down to year 30.

Equation

$$x_{t+1} = 0.05x_t(1 - 0.01x_t) + x_t$$

$x_0 = 10$
$x_1 = 14.5$
Modeling in MS Excel

Equation

\[ x_{t+1} = 0.05x_t \left(1 - 0.01x_t\right) + x_t \]

Now you have estimated population size \( x_t \) from \( t = 0, \ldots, 30 \)
How does a plant population grow in the absence of another species?
How does a second plant species “y” grow in the absence of other species?

The structure of the equation is the same, but we use a different symbol $y_t$ to distinguish from the first species.

$$y_{t+1} = r_y y_t (1 - c_{yy} y_t) + y_t$$
How does a plant grow in presence of a competitor?

- Population growth of plant $x$ also depends on how many plants of species $y$ are present.
- More plants of species $y$ should have a negative effect on species $x$.

$$\Delta x_t = r_x x_t \left(1 - c_{xx} x_t \right)$$
How does a plant grow in presence of a competitor?

• More plants of species “y” should have a negative effect on species “x”

\[ \Delta x_t = r_x x_t \left(1 - c_{xx} x_t \right) \]

\[ \Delta x_t = r_x x_t \left(1 - c_{xx} x_t \right) \]
How does a plant grow in presence of a competitor?

• More plants of species y should have a negative effect on species x

\[ \Delta x_t = r_x x_t \left( 1 - c_{xx} x_t \right) \]

\[ \Delta x_t = r_x x_t \left( 1 - c_{xx} x_t - c_{yx} y_t \right) \]
How does a plant grow in presence of a competitor?

- For species “y” we apply the same rationale and we get

\[ \Delta y_t = r_y y_t (1 - c_{yy} y_t - c_{xy} x_t) \]
Competition dynamics

\[ x_{t+1} = r_x x_t (1 - c_{xx} x_t - c_{yx} y_t) + x_t \]
\[ y_{t+1} = r_y y_t (1 - c_{yy} y_t - c_{xy} x_t) + y_t \]
Lotka-Volterra model for competition

\[
\Delta x_t = r_x x_t (1 - c_{xx} x_t - c_{yx} y_t)
\]

\[
\Delta y_t = r_y y_t (1 - c_{xy} x_t - c_{yy} y_t)
\]
Components of Lotka-Volterra Competition model

How fast a species grows alone

\[
x_{t+1} = r_x x_t (1 - c_{xx} x_t - c_{yx} y_t) + x_t
\]

\[
y_{t+1} = r_y y_t (1 - c_{yy} y_t - c_{xy} x_t) + y_t
\]
Components of Lotka-Volterra Competition model

Competition within species

\[
x_{t+1} = r_x x_t (1 - c_{xx} x_t - c_{yx} y_t) + x_t
\]

\[
y_{t+1} = r_y y_t (1 - c_{yy} y_t - c_{xy} x_t) + y_t
\]
Components of Lotka-Volterra Competition model

Competition between species

\[
x_{t+1} = r_x x_t (1 - c_{xx} x_t - c_{yx} y_t) + x_t
\]
\[
y_{t+1} = r_y y_t (1 - c_{yy} y_t - c_{xy} x_t) + y_t
\]
Modeling in MS Excel

Add another column for species $y$

We will start with 20 plants for species $y$, that is $y_0=20$
Assume the following values for

\( r_x = 0.50 \)
\( r_y = 0.60 \)
\( c_{xx} = 0.01 \)
\( c_{yx} = 0.01 \)
\( c_{yy} = 0.02 \)
\( c_{xy} = 0.005 \)
Modeling in MS Excel

\[ x_{t+1} = r_x x_t (1 - c_{xx} x_t - c_{yx} y_t) + x_t \]
\[ y_{t+1} = r_y y_t (1 - c_{yy} y_t - c_{xy} x_t) + y_t \]

becomes

\[ x_{t+1} = .5x_t (1 - .01x_t - .01y_t) + x_t \]
\[ y_{t+1} = .6y_t (1 - .02y_t - .005x_t) + y_t \]

Insert these formulas in Excel

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<th>year (t)</th>
<th>plants (x)</th>
<th>plants (y)</th>
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</tr>
</tbody>
</table>
Modeling in MS Excel

First, add the term -0.01\(y_t\) in the formula of \(x_{t+1}\) in cell B3

\[
x_{t+1} = 0.5 x_t (1 - 0.01 x_t - 0.01 y_t) + x_t
\]

\[
y_{t+1} = 0.6 y_t (1 - 0.02 y_t - 0.005 x_t) + y_t
\]

= 0.5*B2*(1-0.01*B2-0.01*C2)+B2
Modeling in MS Excel

\[ x_{t+1} = 0.5x_t(1 - 0.01x_t - 0.01y_t) + x_t \]
\[ y_{t+1} = 0.6y_t(1 - 0.02y_t - 0.005x_t) + y_t \]

Type the whole formula for \( y_{t+1} \) in cell C3

\[ C3 = 0.6*C2* (1-0.02*C2-0.005*B2)+C2 \]
Modeling in MS Excel

\[ x_{t+1} = 0.5x_t(1 - 0.01x_t - 0.01y_t) + x_t \]
\[ y_{t+1} = 0.6y_t(1 - 0.02y_t - 0.005x_t) + y_t \]

Select cells B3 and C3 and drag the lower-right corner down to cell C32.
Modeling in MS Excel

\[ x_{t+1} = 0.5x_t(1 - 0.01x_t - 0.01y_t) + x_t \]
\[ y_{t+1} = 0.6y_t(1 - 0.02y_t - 0.005x_t) + y_t \]

Make a scatter plot of these data and see if coexistence is possible.
Stable coexistence of two species

Plants $x_t$ and $y_t$

Year (t)
Stable coexistence of two species
When is coexistence possible?

These equations, (Lotka-Volterra), tell us that coexistence is possible when a species affects its own population growth more than the other species’ population growth.

\[
\begin{array}{c}
0.01 \\
0.02 \\
\end{array}
\begin{array}{c}
c_{xx} > c_{xy} \\
c_{yy} > c_{yx} \\
\end{array}
\begin{array}{c}
0.005 \\
0.01 \\
\end{array}
\]
Why can Buffelgrass outcompete native Sonoran desert plants?

Lotka-Volterra equations state that a plant can invade if the resident species affects its own population growth more than the other invasive species, but the invasive species does not affect its own growth as much.

\[
\begin{align*}
    c_{xx} &> c_{xy} \\
    c_{yy} &< c_{yx}
\end{align*}
\]
Why can Buffelgrass outcompete native Sonoran desert plants?

Lotka-Volterra equations states that a plant can invade if the resident species affects its own population growth more than the other invasive species, but the invasive species does not affect its own growth as much.

\[
\begin{align*}
0.01 & \quad 0.005 \\
0.002 & \quad 0.01
\end{align*}
\]

\[
\begin{align*}
c_{xx} & > c_{xy} \\
c_{yy} & < c_{yx}
\end{align*}
\]
Why can Buffelgrass outcompete native Sonoran desert plants?
Conclusions

1. Competition within species can promote coexistence with other species. This conclusion may solve intriguing ecological puzzles such as why there are more species in the tropics than in northern latitudes.
Conclusions

2. Invasive species may have low levels of within-species competition, which allows them to outcompete native species. This conclusion may be relevant in fighting biological invasions such as the Buffelgrass in the Sonoran desert.
Key points

- Mathematics can provide a deep insight in testing hypothesis in biology (as opposed to a “wet lab” approach).

- If we can mathematically prove that a mechanism may potentially explain certain natural patterns, then it is a valid explanation.
Key points

- Computational thinking provides a whole framework from which high school biology classrooms can benefit:
  - Solving problems: *Why can species coexist?*
  - Designing systems: *Identify the components of population growth in two plant species*
  - Computational concepts: *Recursion, parallel processing* (\(x_t\) and \(y_t\) being computed simultaneously).
  - Abstraction: *Naming species by x and y.*